Observability-Aware Active Calibration of Multi-Sensor Extrinsics

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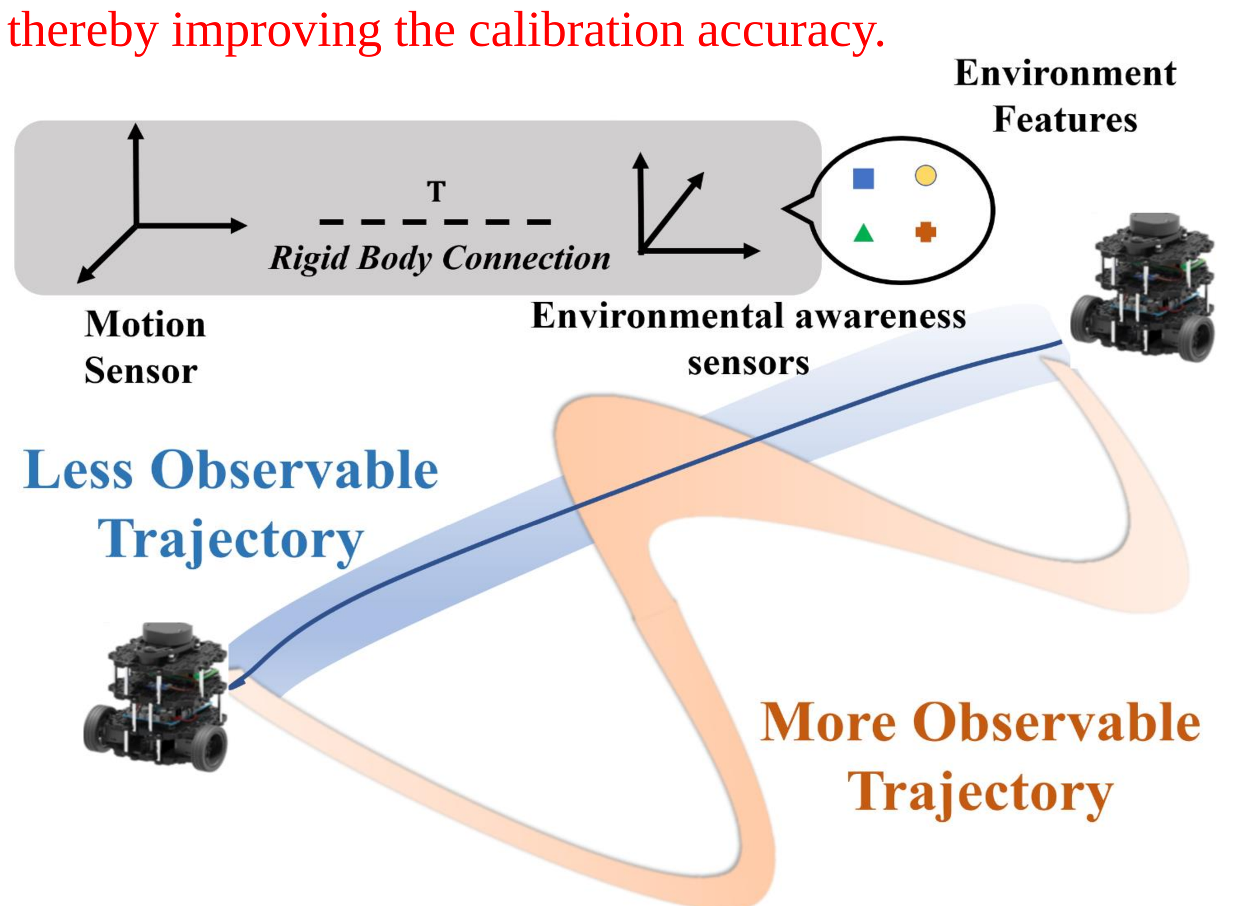
1. Introduction

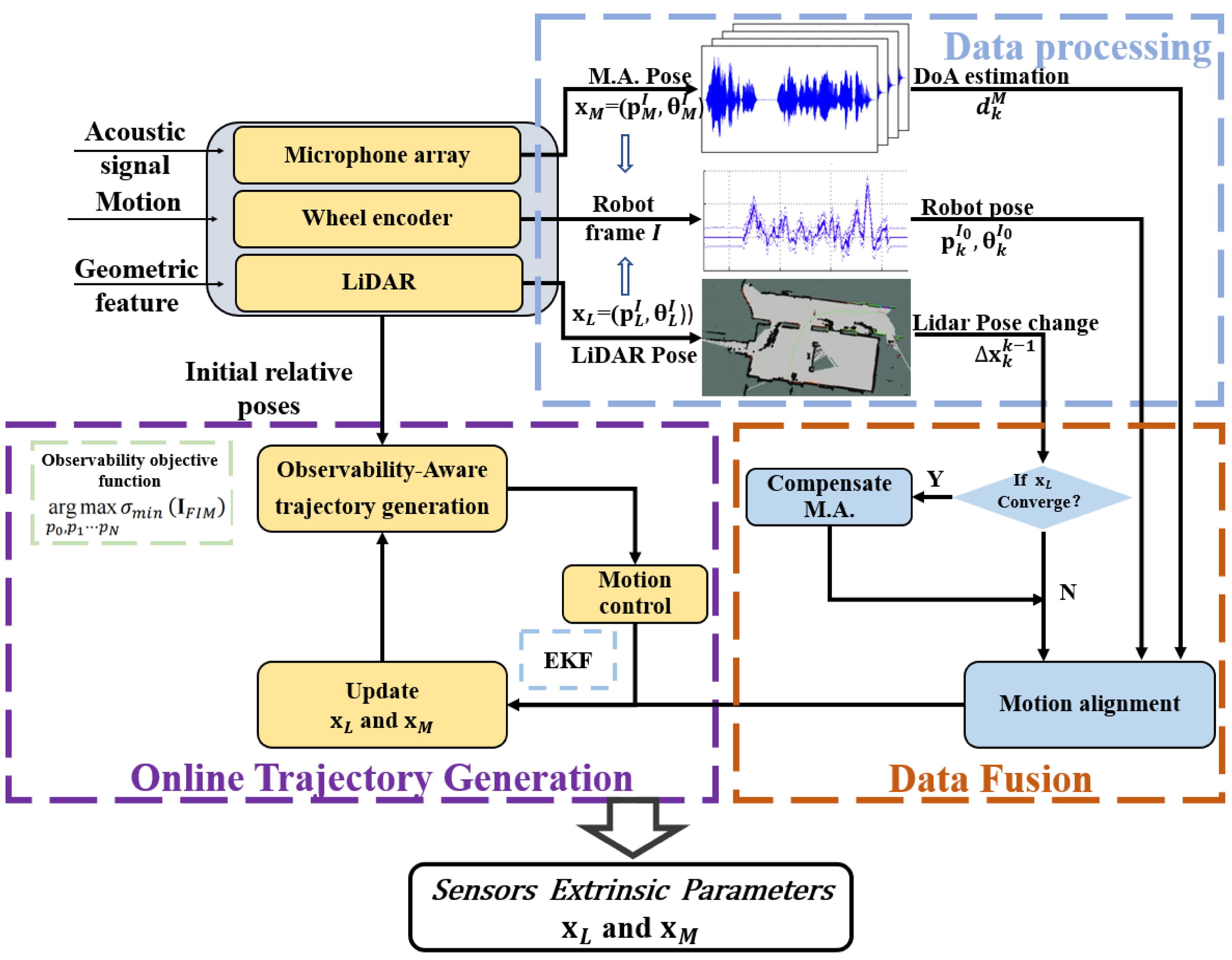
❇ Accurate estimation of sensor extrinsic parameters is fundamental for advanced planning, control, and environmental perception in robotics [1].

❇ Existing methods are mostly off-line in the sense that the robot/source trajectories are manually operated (hence, not optimized) during the data collection process [2,3,4]. Moreover, existing methods seldom consider acoustic sensors, which are important for multi-model perception tasks [5].

❇ For calibrating multiple sensors (Lidar/Microphone array/Wheel encoder) on a robot, here we propose a framework to actively plan the robot trajectory in real time, thereby improving the calibration accuracy.



2. Proposed Method



State vector: $\mathbf{x} = [\mathbf{p}_M^I, \theta_M^I, \mathbf{p}_L^I, \theta_L^I]^\intercal$

Sensors measurement models $\mathbf{h}_k$:

- Wheel encoder:

$$\begin{bmatrix} \mathbf{p}_k^{I_0} \\ \theta_k^{I_0} \end{bmatrix} = \sum_{n=0}^{k} \begin{bmatrix} \cos\theta_n^{I_0} & \cos\theta_n^{I_0} \\ \sin\theta_n^{I_0} & \sin\theta_n^{I_0} \\ -\frac{2}{d_w} & \frac{2}{d_w} \end{bmatrix} \begin{bmatrix} \frac{v_{n\_left}}{2} \\ \frac{v_{n\_right}}{2} \end{bmatrix} \Delta t$$

- LiDAR:

$$\begin{cases} \Delta\mathbf{p}_k^{k-1} = (\mathbf{R}_{k-1}\mathbf{R}_L^I)^\intercal \left(\mathbf{R}_k\mathbf{p}_L^I + \mathbf{p}_k^{I_0} - \mathbf{R}_{k-1}\mathbf{p}_L^I - \mathbf{p}_{k-1}^{I_0}\right) \\ \Delta\theta_k = \theta_L^k - \theta_L^{k-1} \end{cases}$$

- Microphone array:

$$\boldsymbol{d}_k^M = (\mathbf{R}_k\mathbf{R}_M^I)^\intercal \frac{\mathbf{s} - \left(\mathbf{p}_k^{I_0} + \mathbf{R}_k\mathbf{p}_M^I\right)}{\left\|\mathbf{s} - \left(\mathbf{p}_k^{I_0} + \mathbf{R}_k\mathbf{p}_M^I\right)\right\|}$$

2.1 Observability-Aware Trajectory Generation

- **Measure of Observability:**
  Fisher information quantifies the amount of information contained in a set of observations contains about a set of unknown parameters, making it a natural choice for measuring observability [6].
  Fisher information matrix of the expected observation:

$$\mathbf{I}_{\mathrm{FIM}} = \mathbf{J}_\mathbf{x}^\intercal \boldsymbol{\Sigma} \mathbf{J}_\mathbf{x}$$

  $\mathbf{J}_\mathbf{x}$ is the Jacobian matrix of the measurement model w.r.t $\mathbf{x}$.

- **Optimization Objective:**

$$\arg\max_{\boldsymbol{p}} \boldsymbol{\sigma}_{\min}(\mathbf{I}_{\mathrm{FIM}})$$

$\boldsymbol{\sigma}_{\min}$ is the minimal singular value and $\boldsymbol{p}$ is the control points of climped B-spline curve, subject to the robot motion constraints $\boldsymbol{\Omega}$.

2.2 Online Calibration for LiDAR/Acoustic/Motion System

- **Predictor:**

$$\check{\mathbf{x}}_k = \hat{\mathbf{x}}_{k-1},\ \check{\mathbf{P}}_k = \hat{\mathbf{P}}_{k-1}$$

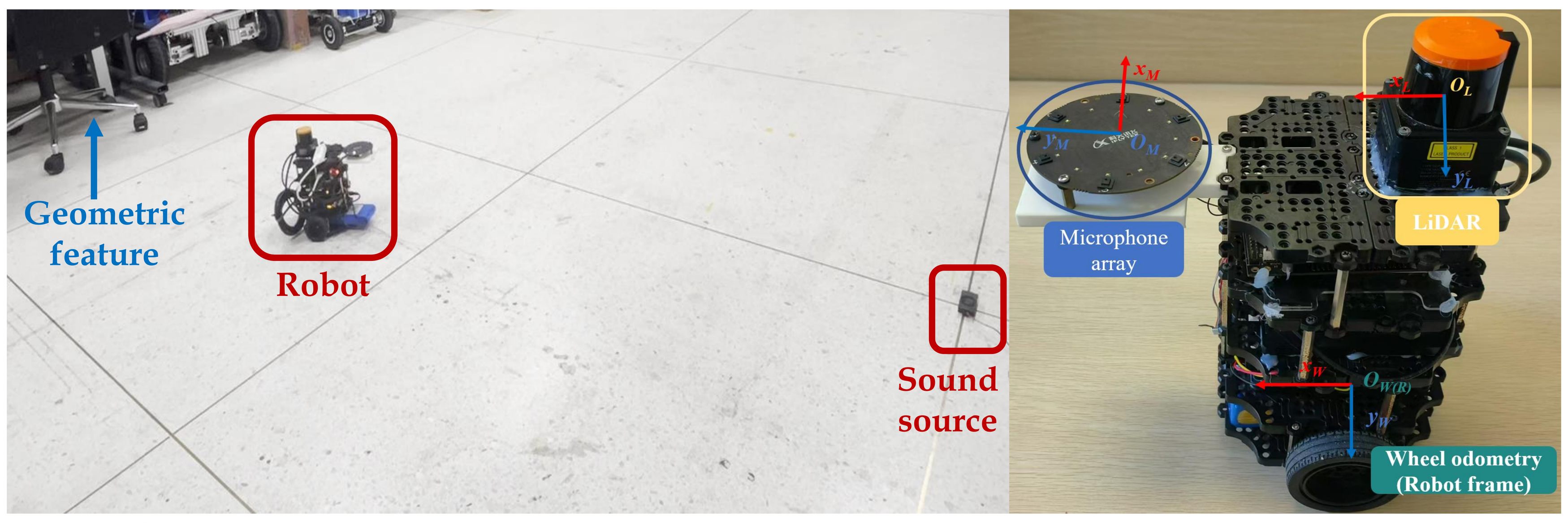
- **Corrector:**

$$\mathbf{K} = \check{\mathbf{P}}_k \mathbf{J}_\mathbf{x}^\intercal(\check{\mathbf{x}}_k)\left(\mathbf{J}_\mathbf{x}(\check{\mathbf{x}}_k)\check{\mathbf{P}}_k\mathbf{J}_\mathbf{x}^\intercal(\check{\mathbf{x}}_k) + \mathbf{R}_k\right)^{-1}$$

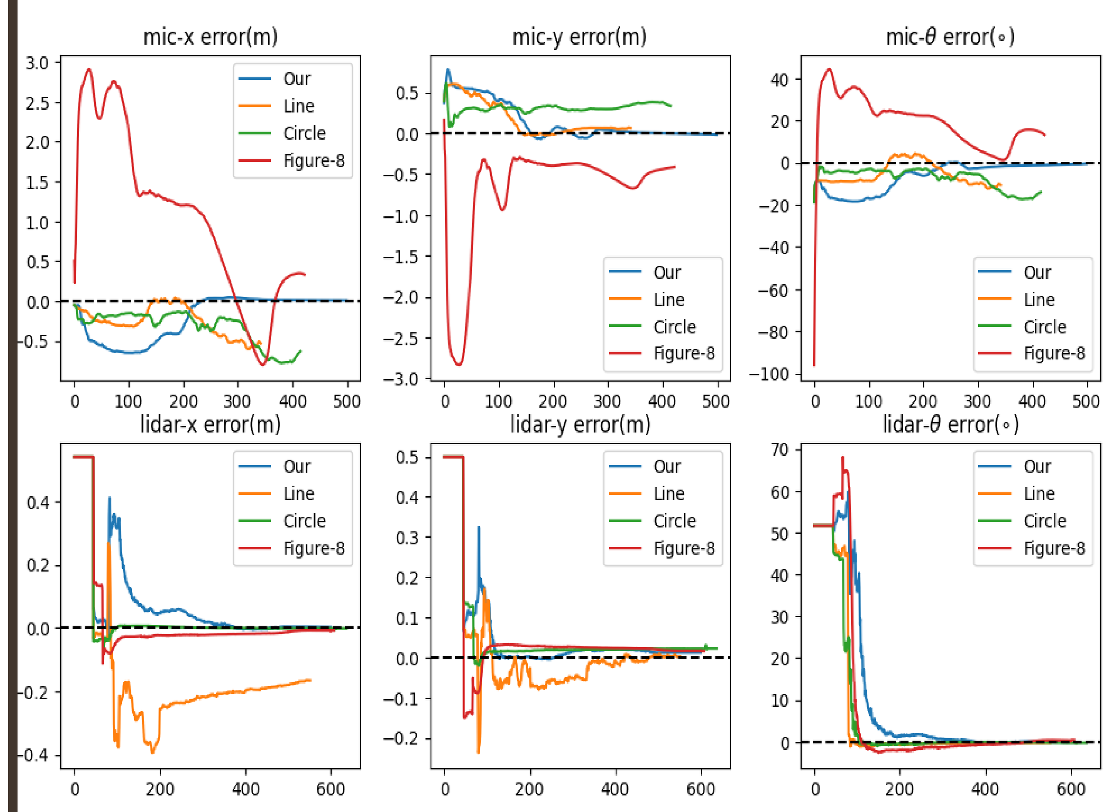
$$\hat{\mathbf{x}}_k = \check{\mathbf{x}}_k + \mathbf{K}(\mathbf{z}_k - \mathbf{h}_k),\quad \hat{\mathbf{P}}_k = \left(\mathbf{I} - \mathbf{K}\mathbf{J}_\mathbf{x}(\hat{\mathbf{x}}_k)\right)\check{\mathbf{P}}_k$$

3. Experiments

- **Experimental Setup:**



- **Calibration Results:**



Microphone array and LIDAR external parameter estimation errors

Traj.	MIC Trans (m)	MIC Orien (°)	LiDAR Trans (m)	LiDAR Orien (°)
Our	**0.013**	**0.585**	**0.015**	0.164
line	1.608	26.332	0.141	**0.114**
circle	0.657	12.879	0.020	0.257
figure_8	0.232	6.019	0.020	0.224

(bold means better)

4. Conclusion

❇ This study proposes an observability-aware active online calibration framework for multi-sensors.

❇ By optimizing the minimum eigenvalue of the Fisher information matrix, the framework generates a trajectory with strong observability using B-spline curves, and updates the sensor extrinsic parameters through Extended Kalman Filtering.

❇ The real-world experimental results demonstrate that the observability-aware active calibration method provides richer excitation to the sensor model and yields more accurate parameter estimates.

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